MTH 310 HW 4 Solutions

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Section 3.1 Problem 10

Is $\{(a, b) \in \mathbb{Z}x\mathbb{Z} | a + b = 0\}$ a subring of $\mathbb{Z}x\mathbb{Z}$?
Answer. No. Since all rings are closed under multiplication, we need only show that if $S = \{(a, b) \in \mathbb{Z}x\mathbb{Z} | a+b = 0\}$, then S is not closed under multiplication. To show this, note that $(1, -1) \in S$ (since 1 + (-1) = 0), but $(1, -1)(1, -1) = (1 * 1, (-1) * (-1)) = (1, 1) \notin S$, since $1 + 1 \neq 0$.

Section 3.2, Problem 3b

Find all idempotent elements in Z_{12}.
Answer. Recall that an element e in a ring is idempotent if $e^2 = e$. Note that $1^2 = 5^2 = 7^2 = 11^2 = 1$ in Z_{12}, and $0^2 = 0$, $2^2 = 4$, $3^2 = 9$, $4^2 = 4$, $6^2 = 0$, $8^2 = 4$, $9^2 = 9$, $10^2 = 4$. Therefore the idempotent elements are 0, 1, 4, iand 9.

Section 3.2, Problem 42

Prove that a finite ring R with identity has characteristic n for some $n > 0$.
Answer. Consider the set $S = \{n*1 | n \in \mathbb{N}\} = \{1,\ 1{+}1,\ 1{+}1{+}1,\ ...\}$, where $\mathbb{N} = \{1, 2, 3, ...\}$. Then since R is closed under (finite) addition, $S \subset R$. But R is finite, so S must be finite as well. In particular, there are some $q, r \in \mathbb{N}$ with $q * 1 = r * 1$. Swapping the values of q and r if necessary, we may assume $q > r$. Then by the distributive property of R we obtain $(q - r) * 1 = 0$. Thus since there is an n with $n * 1 = 0$, there is a smallest n with $n * 1 = 0$, so R has finite characteristic.